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A MODERN CONTROL APPROACH TO GUN FIRING ACCURACY IMPROVEMENTS

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ABSTRACT

The accuracy of firing a gun from a moving combat vehicle is affected significantly by the accuracy with which the gun muzzle is positioned to fire the projectile along a predicted path. Traditionally the problem of accuracy improvement has been addressed through gun mount stabilization while the motion of the barrel is treated as an uncontrolled additional dispersion in the trajectory of the projectile. Modern control technology provides a means of stabilizing the unmeasured gun muzzle. Minimization of a quadratic cost function coupled with estimation of unmeasured states ensure accurate muzzle response to command inputs as well as minimize reaction to base motion and external firing disturbances.

INTRODUCTION

Future fire control systems will be required to exploit the mobility of high performance weapons platforms. The accuracy with which the projectile can be launched along a predicted path must be improved or the demands placed on the gun by modern fire control systems will far exceed its capabilities. Traditionally, weapon control has been accomplished by directing the axis of the gun support gimbal. The gun barrel and therefore the absolute round exit angle is open loop with respect to the axis of rotation. Errors due to barrel bending and induced oscillation are accepted and included in the total firing accuracy budget. The motion of the barrel tip relative to the support gimbal is treated as an uncontrolled additional dispersion in the trajectory of the projectile. It would be desirable to remove most of this dispersion so that each round may be fired in the predicted direction.

Therefore, a need exists to stabilize and control the barrel tip in the presence of external turret motion or firing disturbances. Since useful muzzle position and rate information is difficult to obtain, conventional feedback techniques

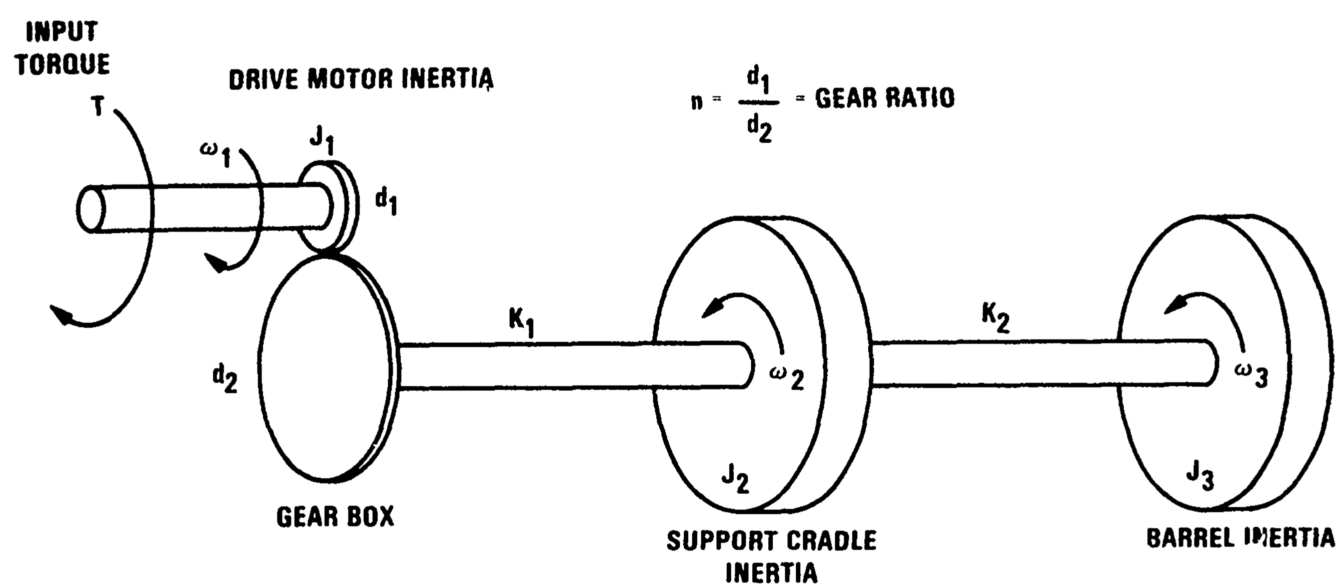
cannot be employed to control the barrel tip. The problem, then, is to use modern control theory employing state variable feedback and to supply the necessary parameter information through state estimation. In the approach presented here, the control emphasis has been placed on the barrel tip angular rate rather than position to ease the demands on the state estimator and provide the fastest command response and disturbance rejection. An optimal control technique with control tuning is employed to achieve the best response while maintaining adequate margins for model imperfections and estimator lag.

Effective state estimation depends on the degree of model fidelity and a model definition which allows the observation of every state via available measurements. Therefore, to impose a practical challenge to an otherwise academic approach, an Emerlec-30 (a twin 30 mm naval gun mount produced by Emerson Electric Co.) was chosen as the control test bed for the barrel tip optimal control and estimation technique. The only practical implementation of this technique, which requires the execution of numerous recursive equations in real time, demands the speed, memory, and flexibility of a digital processor. Therefore, the technique is configured such that all processing can be executed in a standard micro or minicomputer with serial or parallel I/O.

This paper presents the results of applying an optimal control and state estimation technique to achieve control over the barrel tip angular rate of a 30 mm gimballed gun mount. Simulation results are derived from a model of the test mount with all of its parameter characteristics and limitations.

TECHNICAL APPROACH

The general approach to this project has been to use optimal control and estimation techniques to achieve muzzle stabilization and control of a test bed gimballed gun mount (an Emerlec-30). Since a good optimal controller and an effective state estimator depend on model fidelity, the first step was to develop a representative math model of the gun mount plant using test data. The Emerlec-30 contains an electric motor drive geared to each control axis of the gun barrel. Frequency response data was obtained between various points from motor command to barrel tip by using a dual channel spectrum analyzer and rate gyros. Since high order estimators result in unwieldy algorithms for microprocessor implementation, a practical goal in model development was to restrict the order to as low as practical and still model the dominant characteristics. The frequency response data suggested that a series three body, fifth order model, such as shown in Figure 1, was adequate.



$$\omega_1/T = [J_2J_3S^4 + (K_1J_3 + K_2J_2 + K_2J_3)S^2 + K_1K_2]/\Delta$$

$$\omega_2/T = (n\,J_3K_1S^2 + n\,K_1K_2)/\Delta$$

$$\omega_3/T = n\,K_1K_2/\Delta$$

$$\Delta = J_1J_2J_3S^5 + (K_1J_1J_3 + n^2\,K_1J_2J_3 + K_2J_1J_3 + K_2J_1J_2)S^3 + K_1K_2(J_1 + n^2J_2 + n^2J_3)S$$

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Figure 1 Series Three-Body Plant Model

Using this basic model, parameters were calculated and adjusted, and damping terms were added until a very good frequency response match was obtained. Because the math model is fundamental to the real time estimator and is a key element in the development of the optimal control gains, its fidelity as a model of the actual plant is important. The more model imperfections allowed, the greater the control margins necessary for successful performance and a more suboptimal controller results. In addition, modeling imperfections cause unwanted estimator error dynamics that usually add more phase shift to the controller loop.

After defining the math model of the plant, a suitable sampling frequency is chosen to adequately control the highest frequency of interest through a digital computer. A sampling rate of 100 Hertz was chosen for good control out to the firing frequency of 10 Hertz. The continuous plant frequency domain definition was then transformed to the digital time domain for use in the control and estimation algorithms. This transformation was accomplished on a computer by a truncated series expansion of the system state transition matrix.

OPTIMAL CONTROL DEVELOPMENT

The word "optimal" in its general sense means "best" or "most desirable." In controls, as in other endeavors, the engineer is always searching for the best solution to the problem at hand. Via classical techniques (Nyquist, Nichols, Bode, root locus, etc.) the control problem concerns itself with finding "optimum controller settings", which is a process of parameter optimization. In optimal control techniques, the problem is concerned with finding the best control strategy for a given system and cost function. In determining an optimal control strategy, no prior assumptions or commitments are made that would fix the controller structure; on the other hand, in parameter optimization, the problem is to first fix the structure of the controller (e.g., a proportional-plus-integral controller, etc.) and then attempt to determine the optimum parameters. The aim of optimal control is to find some policy, which, when applied to the system input, will optimize the system's performance with respect to some cost function.

Although the variational viewpoint is the basis from which the engineer seeks an optimal strategy, its mathematical difficulty and computational complexity are so great that real progress in the field started only after major breakthroughs by Bellman (dynamic programming, 1960) and Pontryagin (maximum principle, 1962). Since then, the problem has attracted the attention of

many control mathematicians who have offered formal, rigorous, and general, but often very abstract, contributions. Even though engineers can often obtain solutions only by using simplified system models and simple performance indices, the optimal control approach has been an important tool, for example, by pointing the way to better controller structures, or by providing a measure of performance so that a designer can determine whether there is any sizable room for improvement in his design.

The approach taken to achieve control over virtually an unmeasurable entity, the barrel tip of a gimballed 30 mm gun mount, was to employ optimal control theory to define the control strategy and then identify and circumvent the problem areas with classical techniques. A parameter optimization of the resulting hybrid system then results in the best solution that can be practically implemented.

The control scheme chosen was a variation on R. Bellman's dynamic programming with modifications suggested by various standard control textbooks. The scheme is based on the principle of optimality, which states that an optimal policy (best system input sequence) has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. The criterion for optimality was the minimization of a quadratic cost function. This function combines the sum of the squares of the errors between the actual system states and desired states plus the sum of the squares of system input values. The latter term keeps the control policy feasible by penalizing excessively large input values (i.e., infinite energy cannot be supplied to the system via the controller). As with most criteria, they are not all-encompassing. Achieving a minimum error in a certain desired state may not assure good step responses, frequency responses, or system disturbance immunity. Therefore, the cost function is adjusted through the values of an error state weighting matrix until the optimal policy derived from cost function minimization results in desired system response.

To illustrate the technique, consider the continuous system described by the differential equation:

$$\dot{x}(t)] = [A]x(t)] + B]u(t)$$

where x(t)] is the system state column vector, u(t) is the

input, [A] and B] are coefficient matrices. The z-transform of this system results in the state transition equation:

$$x(n+1)] = [\Phi(T)]x(n)] + \theta(T)]u(n)$$

where T is the sample interval and

$$[\Phi(T)] = e^{[A]T},$$

$$\theta(T)] = \int_0^T [\Phi(T-\lambda)]B]d\lambda.$$

Then, if the desired states of the system can be described by

$$x_d(n+1)] = [\Psi(T)]x_d(n)],$$

error states can be defined by

$$y(n)] = x_d(n)] - x(n)].$$

An error state transition equation can then be defined by

$$y(n+1)] = [\Gamma(T)]y(n)] + \Omega(T)]u(n)$$

where $[\Gamma(T)]$ is a function of $[\Phi(T)]$ and $[\Psi(T)]$, and $\Omega(T)]$ is a function of $\theta(T)]$. An optimal control policy would then be a set of input commands, u(n), that drive the error states in such a way that a certain performance criterion is exactly satisfied. This optimal policy should have the form

$$u(n) = H(T)]' y(n)]$$

where $H(T)]'$ is the transpose of the optimum gain column vector, H(T)]. This equation suggests that control can be achieved through state variable feedback.

To continue the design, a suitable performance criterion must be selected. Since the system is defined with error states, initially the best policy would be that which drives the errors to zero. In essence, the minimization of the sum of the square of the errors should suffice. Therefore, a quadratic performance index (or cost function) was defined by

$$I = \sum_{n=1}^{N} y(n)]'[S]y(n)] + \lambda u^2(n-1)$$

where [S] is the error weighting matrix, λ is an input penalty factor, and N is the last stage of an N-stage process. Using the principle of optimality, and working from the Nth stage backward to the start of the process, the optimum gain vector is developed:

$$H(NT-jT)]' = - \frac{\Omega(T)]'\{[S] + [P(NT-jT-T)]\}[\Gamma(T)]}{\Omega(T)]'\{[S] + [P(NT-jT-T)]\}\Omega(T)] + \lambda}$$

where j = 0, 1, 2, ..., N-1, and

$$[P(NT-jT)] = [\Gamma(T)]'\{[S] + [P(NT-jT-T)]\}$$
$$* \{[\Gamma(T)] + \Omega(T)]\ H(NT-jT)]'\}$$

These equations are solved recursively starting with j=N-1 and

$$[P(NT-jT-T)] = [P(0)] = 0.0$$

For an infinite stage process, the indicated iteratious can be carried until the optimum gain vector values H(NT-jT)]', reach a steady state. These steady state values are then used in a fixed-gain feedback control configuration in which the only variables affecting the optimal policy are the error states. Since the optimal control policy will always result in the minimization of the cost function, changes in system performance can be effected by changing the cost function. This is accomplished by adjusting the error weighting matrix [S], and the input penalty factor, λ. Adjustments are made until the desired performance is achieved.

While the optimal control technique is an excellent way of handling a multiple feedback control law design, it does not automatically build in stability margins to allow for model imperfections or estimator lag. Therefore, using the results of the optimal control law development as a base system, each subloop was analyzed and tuned as necessary to provide adequate stability margin. Additional dynamics were included as necessary when the best control could not be achieved with a simple gain feedback. A block diagram of the resulting system is shown in Figure 2. This control law tuning is an exercise in classical control law design but retains the optimal control configuration. Optimal control tuning is the key to reliable and accurate pointing of the barrel tip. The best performance characteristics of the optimal controller are retained, as evidenced in the Figure 3 frequency response of barrel tip versus input command, while added stability margin assures successful implementation.

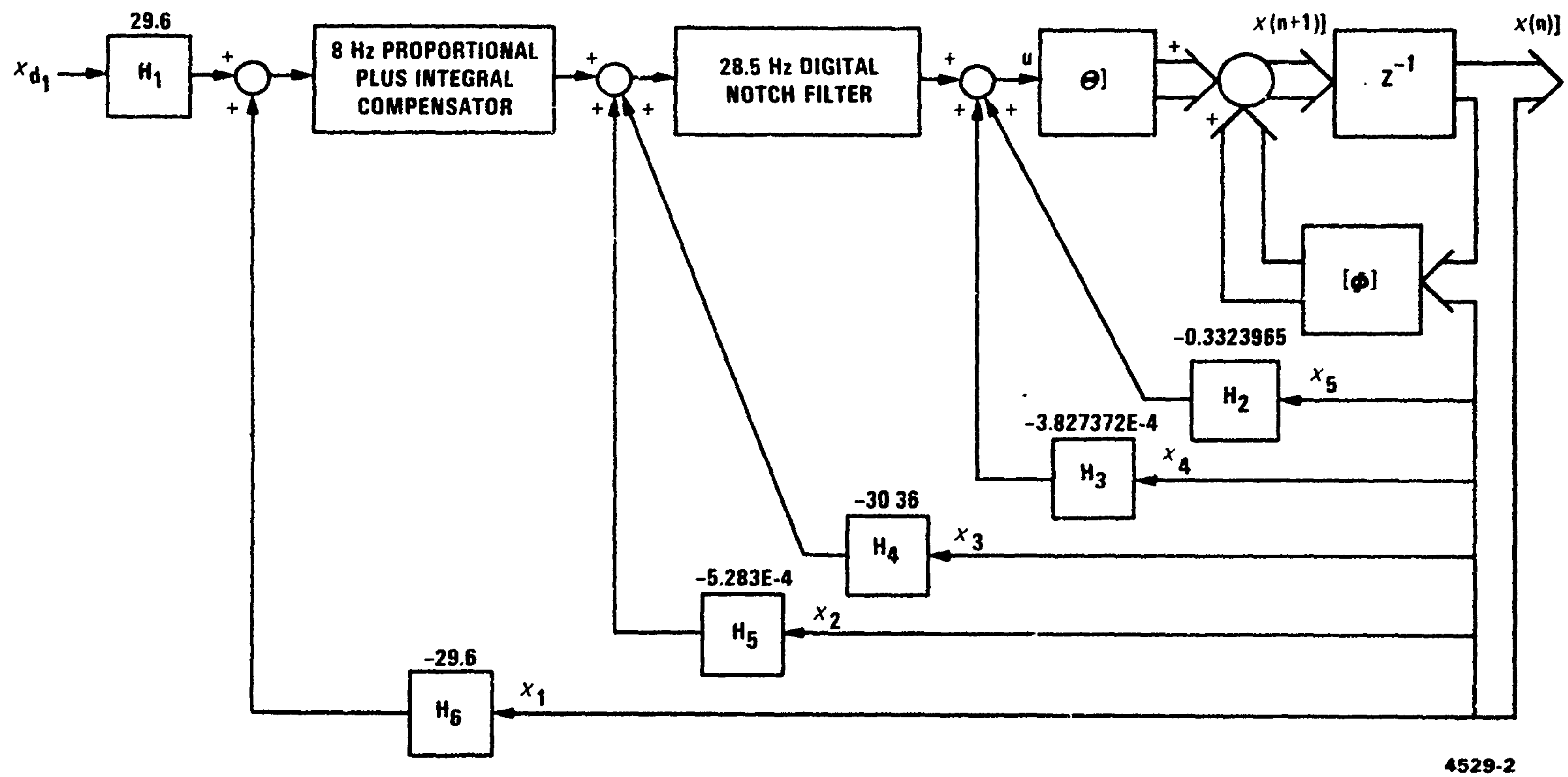

Figure 2 Block Diagram of Optimal Controller

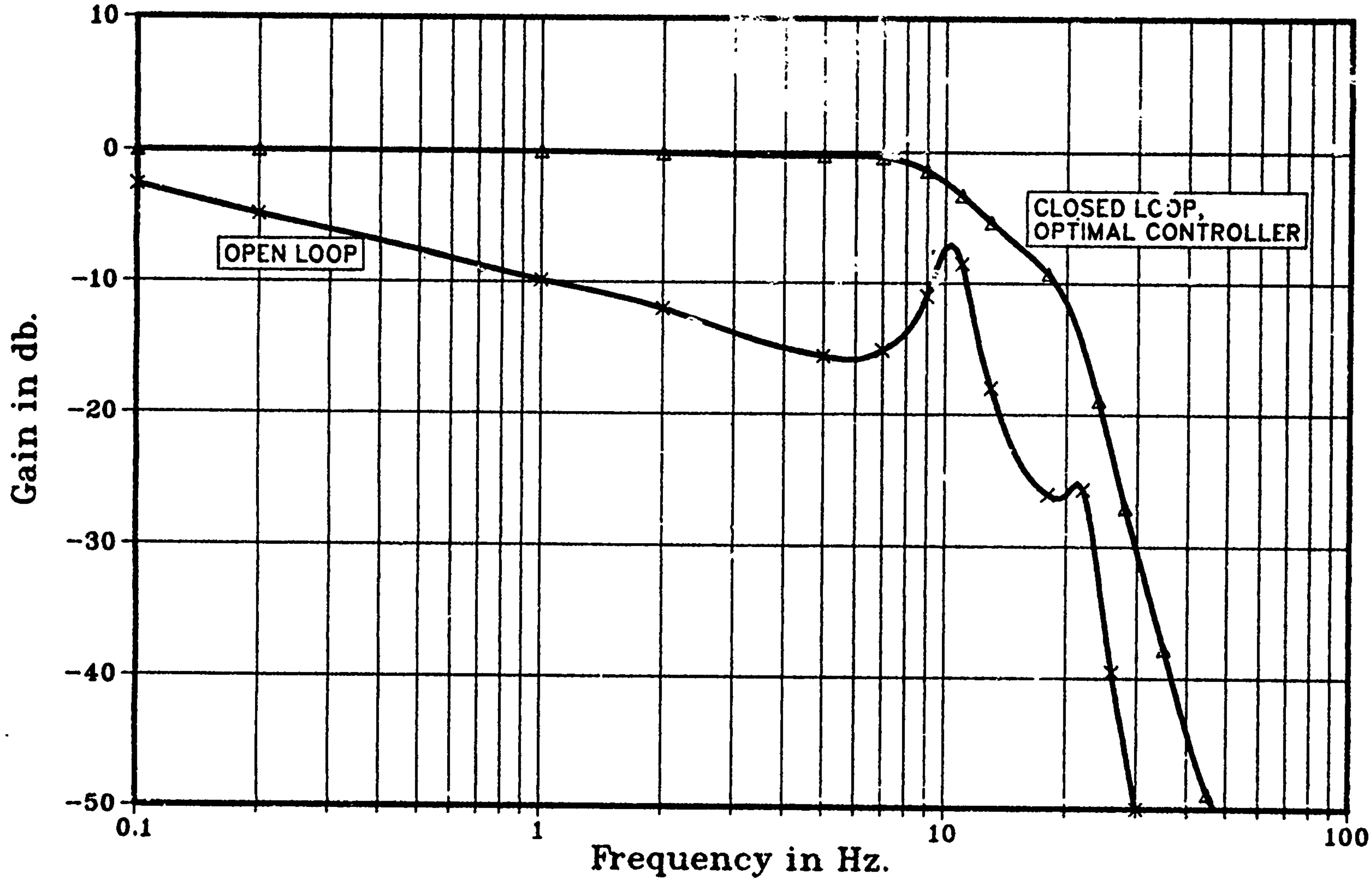

Figure 3 Muzzle Rate vs. Input Command

Besides a performance goal of good input command response out to the 10 Hertz firing frequency, an essential goal for stabilization is good rejection of vehicle disturbances. The highest dominant rate disturbance frequency expected from the vehicle in typical applications is 1 Hertz. Therefore, at least 90% attenuation of all disturbances up to 1 Hertz was a primary consideration during control law development. As shown in Figure 4, this goal, and more (greater than 20 db attenuation below 1 Hertz), was achieved. A time history of the barrel tip response to a 1 Hertz, 10 deg/sec vehicle rate disturbance is shown in Figure 5.

ESTIMATOR DEVELOPMENT

With an approach directed at control over an unmeasured system state, a state estimator is not only necessary but becomes the fundamental backbone of the control law. The accuracy with which the barrel tip can be pointed by the controller depends primarily upon the accuracy with which the barrel tip state variable can be estimated. A simple predictor type estimator, based solely on the propagation of plant input commands through a math model, derives its accuracy from the fidelity of the model. However, perfection is seldom achieved. A typical gun mount is difficult to precisely model due to complex structural interaction, mount-to-mount production variations, and day-to-day environmental changes resulting in friction and gear backlash variations. Therefore, in a practical application, the predictor type estimator will usually fail.

A predictor-corrector type estimator, based on feedback control concepts, offers a viable solution to the estimation problem. Although this type estimator is not without its problems, it is more immune to modeling imperfections than the essentially open loop predictor. The predictor-corrector propagates all inputs and disturbances through a math model the same as a simple predictor. A correction to the predicted states is then made based on the error between measurements of actual states and their estimated counterparts. Modeling imperfections, instead of causing estimate divergence as encountered in the predictor, results in varying degrees of underdamped estimate performance depending on the degree of imperfection and the gains associated with the correction. Therefore, the choice of a correction gain vector, L], must be matched to the expected modeling imperfection and plant dynamics.

A somewhat painless approach to determining L] is to employ Ackermann's formula. This formula allows the placement of estimator characteristic roots anywhere in the z-plane unit

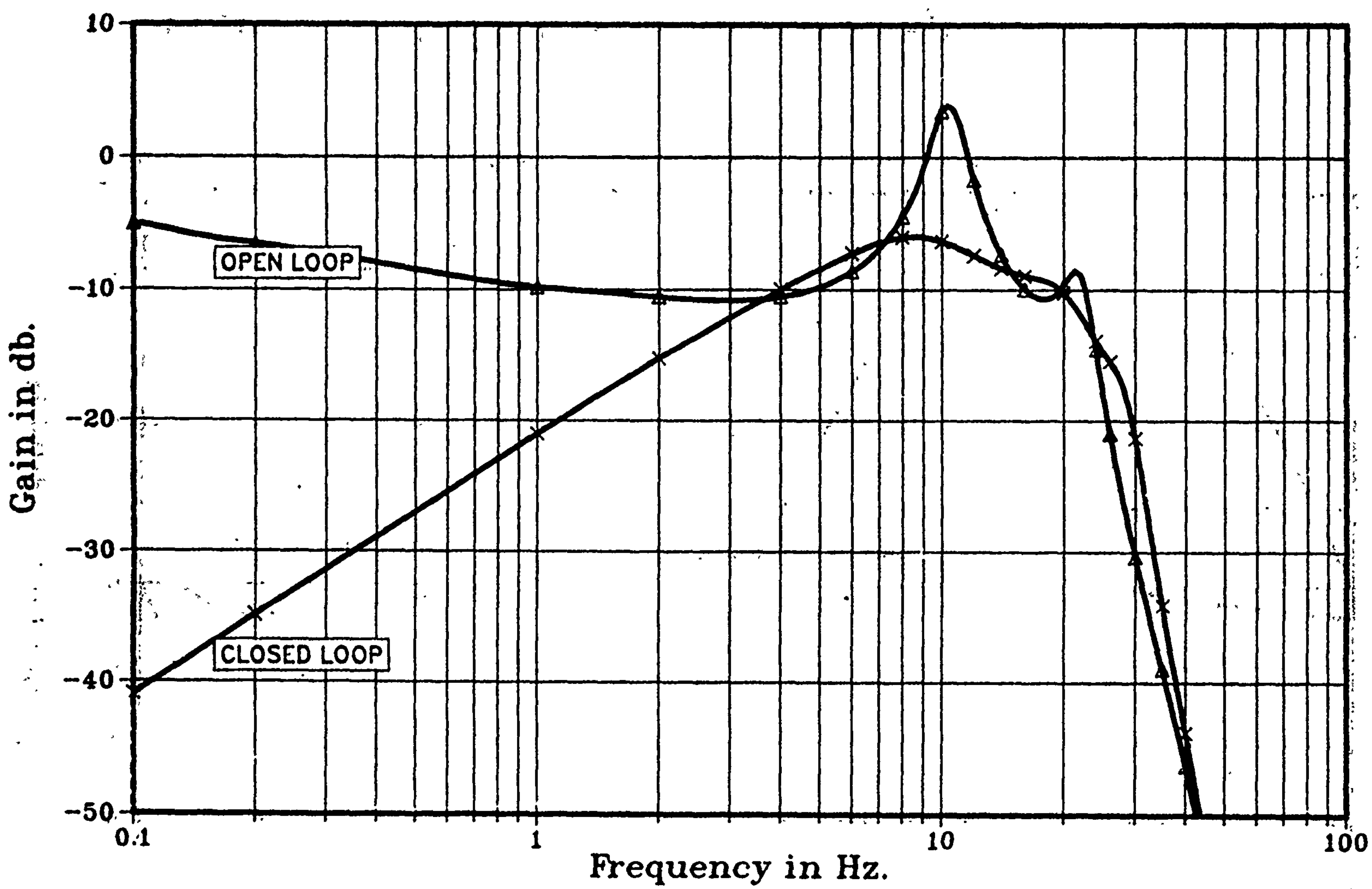

Figure 4 Muzzle Rate vs. Vehicle Disturbance

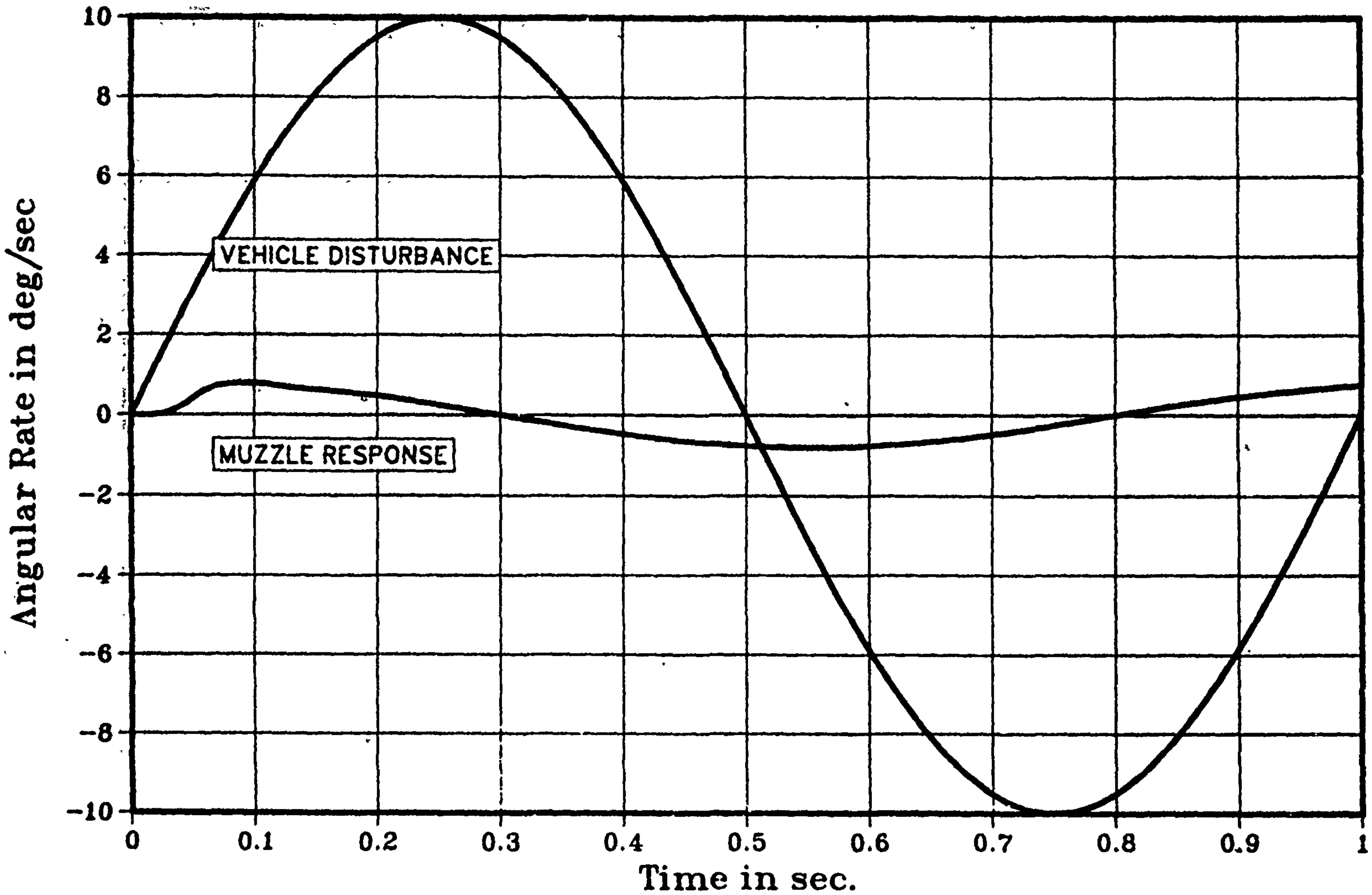

Figure 5 1 Hz. Disturbance Response

circle. By placing estimator poles in desired locations, a desired characteristic equation will result. This equation can be represented by:

$$\alpha_e(z) = z^k + \alpha_1 z^{k-1} + \alpha_2 z^{k-2} + \ldots + \alpha_k,$$

where k is the order of the system. Then, using the state transition matrix of the system model, $[\Phi(T)]$, a matrix characteristic equation can be constructed:

$$\alpha_e([\Phi(T)]) = [\Phi(T)]^k + \alpha_1[\Phi(T)]^{k-1} + \ldots .\alpha_k \quad .$$

This equation is dual with $\alpha_e(z)$. Finally, with a vector relating the measurement to the measured states, $D]'$, the Ackermann formula allows $L]$ to be determined by:

$$L] = \alpha_e([\Phi(T)]) \begin{bmatrix} D]'[\Phi(T)] \\ D]'[\Phi(T)]^2 \\ \cdot \\ \cdot \\ \cdot \\ D]'[\Phi(T)]^k \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$

Although painless, this approach allows pole-zero cancellation to obtain the desired characteristics. As such, the resulting estimator feedback system is very sensitive to a mismatch between the actual plant and the math model. This sensitivity is manifest in undesired estimated state dynamics. To avoid the difficulties, the correction gain vector, L], is adjusted via classical techniques so that each estimate displays a fast, damped response to system measurement.

In order to finish the fine tuning of L], the complete estimator must be considered. A block diagram of the estimator, and its relationship to the entire system, is shown in Figure 6. Since measured states include anti-aliasing filter and sensor dynamics, they are somewhat different from the plant states that are to be controlled. These differences must be accounted for in the estimator definition.

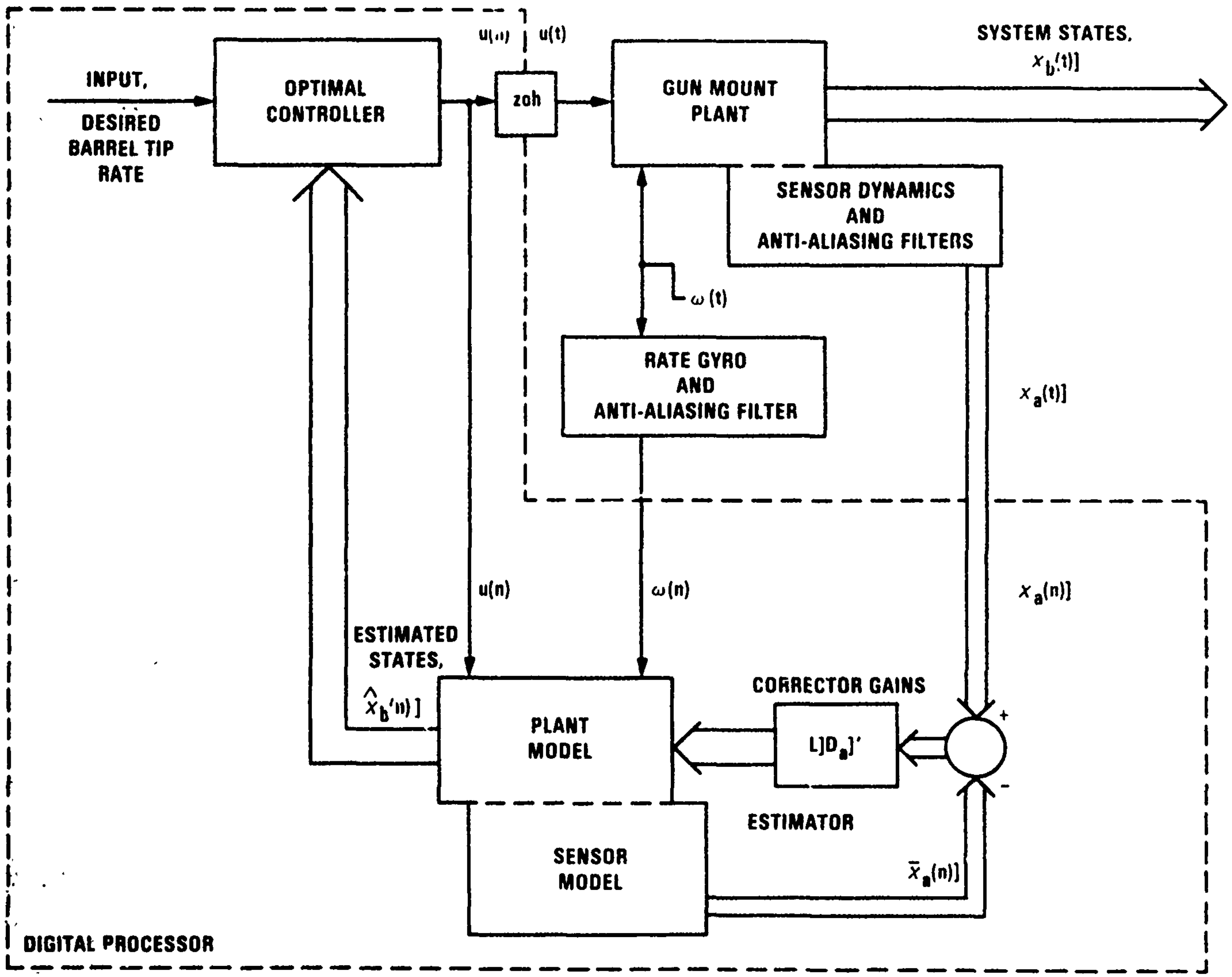

Figure 6 Barrel Tip Control System

Since the measured states are adequately filtered for anti-aliasing and do not need to be estimated, a reduced-order estimator can be constructed by partitioning the state transition equation into measured and unmeasured states:

$$\begin{bmatrix} x_a(n+1)] \\ x_b(n+1)] \end{bmatrix} = \begin{bmatrix} [\Phi_{aa}] & [\Phi_{ab}] \\ [\Phi_{ba}] & [\Phi_{bb}] \end{bmatrix} \begin{bmatrix} x_a(n)] \\ x_b(n)] \end{bmatrix} + \begin{bmatrix} \Theta_a] \\ \Theta_b] \end{bmatrix} u(n) + \begin{bmatrix} \Psi_a] \\ \Psi_b] \end{bmatrix} w(n)$$

where $x_a(n)]$ is the measured states, $x_b(n)]$ the unmeasured states, u(n) the plant input, and w(n) is the vehicle disturbance. An associated measurement equation can be described by:

$$y(n)] = \begin{bmatrix} D_a]' & x_a(n)] \\ 0 & x_b(n)] \end{bmatrix}$$

Using the partitioned definition of the system, the estimator algorithm is described by the following three equations:

$$\bar{x}_b(n+1)] = [\Phi_{bb}]\hat{x}_b(n)] + [\Phi_{ba}]x_a(n)] + \Theta_b]u(n) + \Psi_b]w(n) \ ,$$

$$\bar{x}_a(n+1) = [\Phi_{aa}]x_a(n)] + [\Phi_{ab}]\bar{x}_b(n+1)] + \Theta_a]u(n) + \Psi_a]w(n) \ ,$$

$$\hat{x}_b(n+1)] = \bar{x}_b(n+1)] + L]D_a]'\{x_a(n+1)] - \bar{x}_a(n+1)]\},$$

where $\bar{x}]$ is an intermediate prediction of the measured and un- measured states and $\hat{x}]$ is the corrected estimate. The esti-

mator equations are executed in the order given above. The first equation is simply a propagation of the unmeasured estimator forward in time through the state transition matrix (model of the plant). Included in the equation are the inputs, u(n), the vehicle disturbance, w(n), and the measured states, $x_a(n)]$. The second equation is similar to the first, but predicts the future measured states. Finally, the third equation is calculated at the time N-1 so that a comparison between the predicted measured states and actual measured states can be made to determine the error in the estimate. This error, through the gain vectors L] and $D_a]$, is used to correct the prediction resulting in the best estimate of unmeasured states, $\hat{x}_b]$.

Although the estimator can be expected to add some phase lag to the optimal control feedback states, a well designed estimator will cause minimal degradation to the overall performance. A simulation step response of the Emerlec-30 test gun mount is shown in Figure 7. The excellent performance displayed by the tuned optimal controller was only slightly modified by the addition of the estimator. More importantly, control over a critical unmeasured system state, the barrel tip rate, was achieved.

SUMMARY

A modern control approach to firing accuracy improvements has been developed for a 30 mm naval gun mount. Optimal control techniques were applied to achieve the best control over the barrel tip while meeting the desired performance criteria of quick, damped response to system rate inputs and attenuation of system disturbances. The practical application of the optimal controller to a 30 mm test bed gun mount was made feasible by employing control tuning based on classical control techniques. Without tuning, the stability margins obtained through optimal control design would be inadequate for any imperfections in state estimation or plant modeling.

The key to achieving control over the barrel tip rate, since the useful measurement of tip rate information for feedback control is not only difficult but expensive, lies in the successful estimation of this system state. Using feedback control techniques, a predictor-corrector type state estimator was developed with minimal (no more than 15 deg) phase lag added to each state control loop.

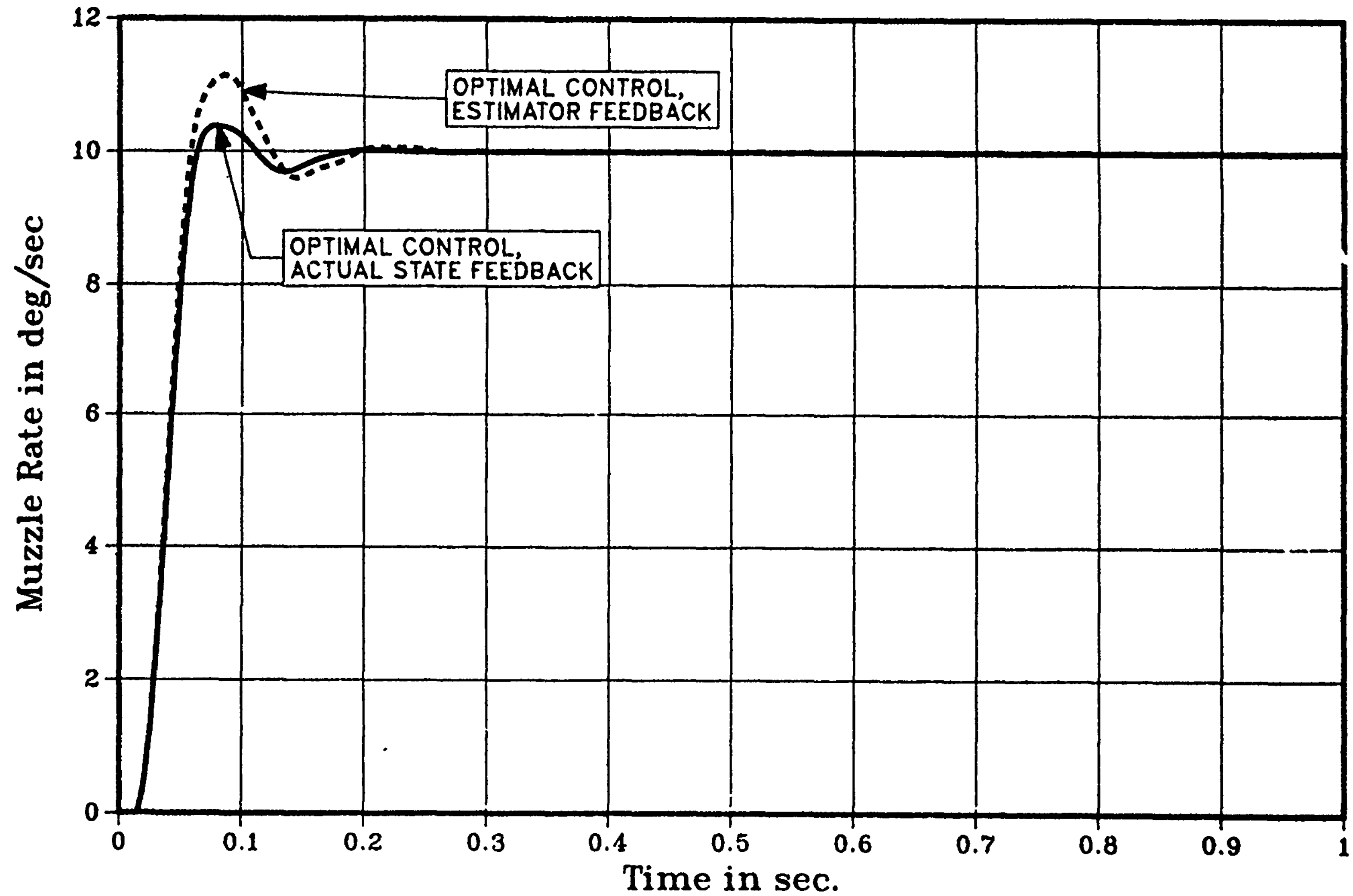

Figure 7 System Step Response

Together the tuned optimal controller and the predictor-corrector state estimator embody a modern state variable control technique to stabilize and drive the gun barrel tip. Control over the tip (or muzzle) rate greatly improves the pointing accuracy thereby decreasing the dispersion encountered when a gun is fired from a moving vehicle. By including higher order modes of the barrel and gun mount, the same technique could be extended to control virtually any point along the barrel. Thus, future firing accuracy may only be limited by the projectile itself.

The next phase of this endeavor will be to code the optimal controller and estimator for microprocessor control of the test gun barrel tip. Pointing accuracy will be assessed during various laboratory controlled disturbances. Also, a test firing should provide a real disturbance environment and penultimate proof of concept. The final proof should be in the field in battle conditions.

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